KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY
DEPARTMENT OF COMPUTER ENGINEERING

COE 272
Digital Systems

Lecture 3: Boolean Algebra and Binary Logic (Part 2)

What are Karnaugh[1] maps?

- Karnaugh maps provide an alternative way of simplifying logic circuits.
- Instead of using Boolean algebra simplification techniques, you can transfer logic values from a Boolean statement or a truth table into a Karnaugh map.
- The arrangement of 0's and 1's within the map helps you to visualise the logic relationships between the variables and leads directly to a simplified Boolean statement.

[1]Named after the American electrical engineer Maurice Karnaugh.

Karnaugh maps

- Karnaugh maps, or K-maps, are often used to simplify logic problems with 2, 3 or 4 variables.

Cell = 2^n ,where n is a number of variables

For the case of 2 variables, we form a map consisting of 2^2=4 cells as shown in Figure

B \ A	0	1
0	$A+B$	$\overline{A}+B$
1	$A+\overline{B}$	$\overline{A}+\overline{B}$

Maxterm

B \ A	0	1
0	00 (0)	10 (2)
1	01 (1)	11 (3)

B \ A	0	1
0	$\overline{A}\overline{B}$	$A\overline{B}$
1	$\overline{A}B$	AB

Minterm

Karnaugh maps

- 3 variables Karnaugh map

Cell = 2^3=8

C \ AB	00	01	11	10
0	0: $\overline{A}\,\overline{B}\,\overline{C}$	2: $\overline{A}B\overline{C}$	6: $AB\overline{C}$	4: $A\overline{B}\,\overline{C}$
1	1: $\overline{A}\,\overline{B}C$	3: $\overline{A}BC$	7: ABC	5: $A\overline{B}C$

Karnaugh maps

- 4 variables Karnaugh map

CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

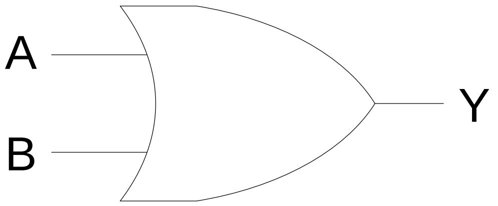
Karnaugh maps

- The Karnaugh map is completed by entering a '1‘(or ‘0’) in each of the appropriate cells.

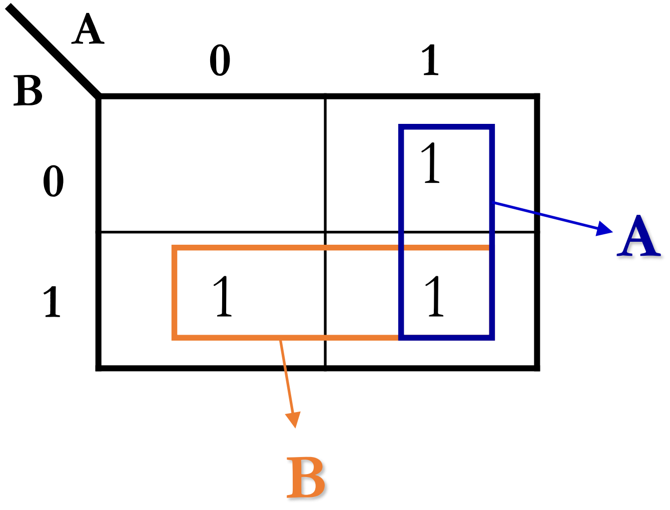
- Within the map, adjacent cells containing 1's (or 0’s) are grouped together in twos, fours, or eights.

Example

2-variable Karnaugh maps are trivial but can be used to introduce the methods you need to learn. The map for a 2-input OR gate looks like this:



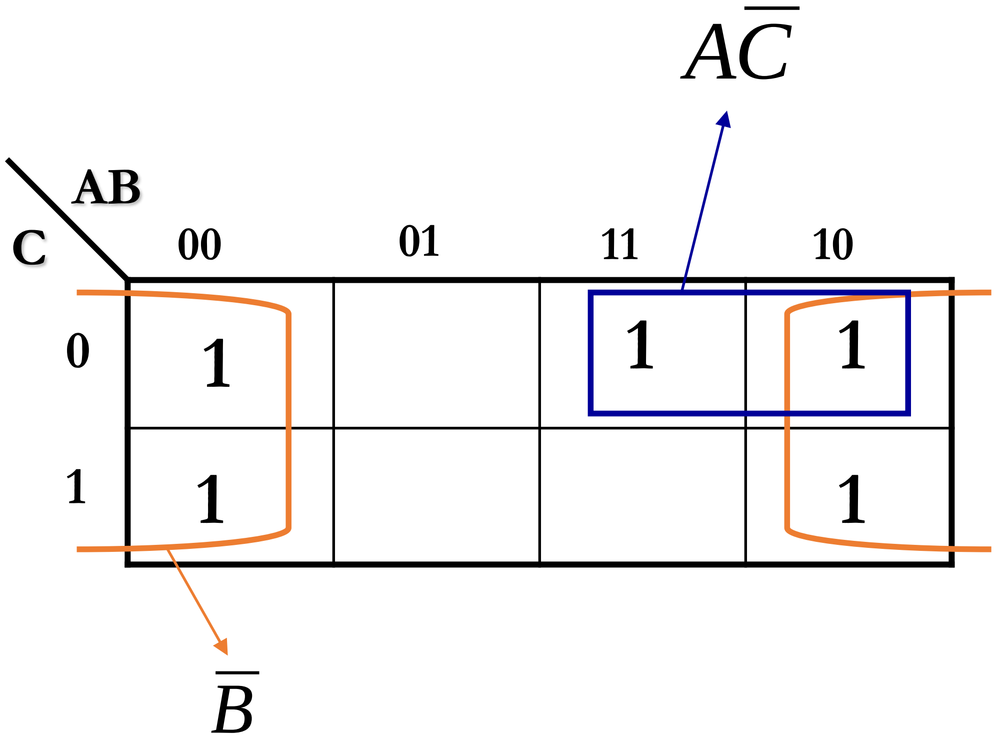
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1



A+B

Example

A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

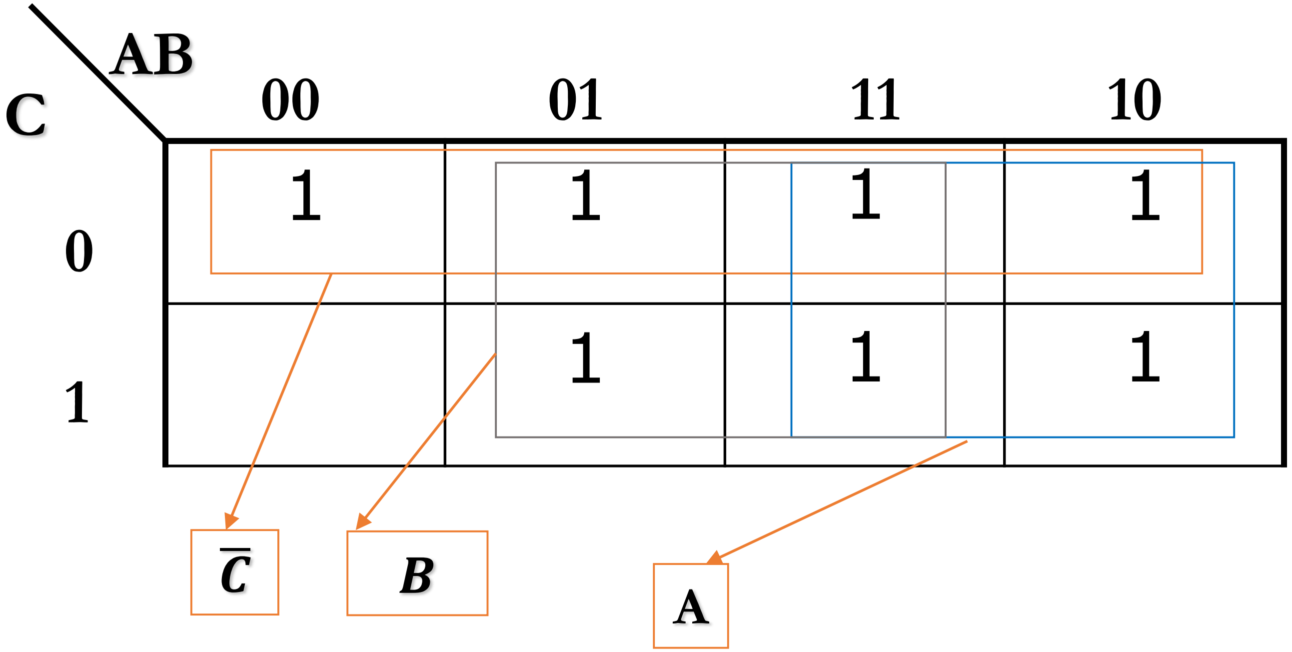


$$\overline{B} + A\overline{C}$$

Exercise

- Let us use Karnaugh map to simplify the follow function.
  1. $F(A,B,C) = m_0+m_2+m_3+m_4+m_5+m_6+m_7$
  2. $Y = m_0+m_1+m_2+m_5+m_7$

- Answer (1)



$F(A, B, C) = \boldsymbol{B} + \mathbf{A} + \boldsymbol{\overline{C}}$

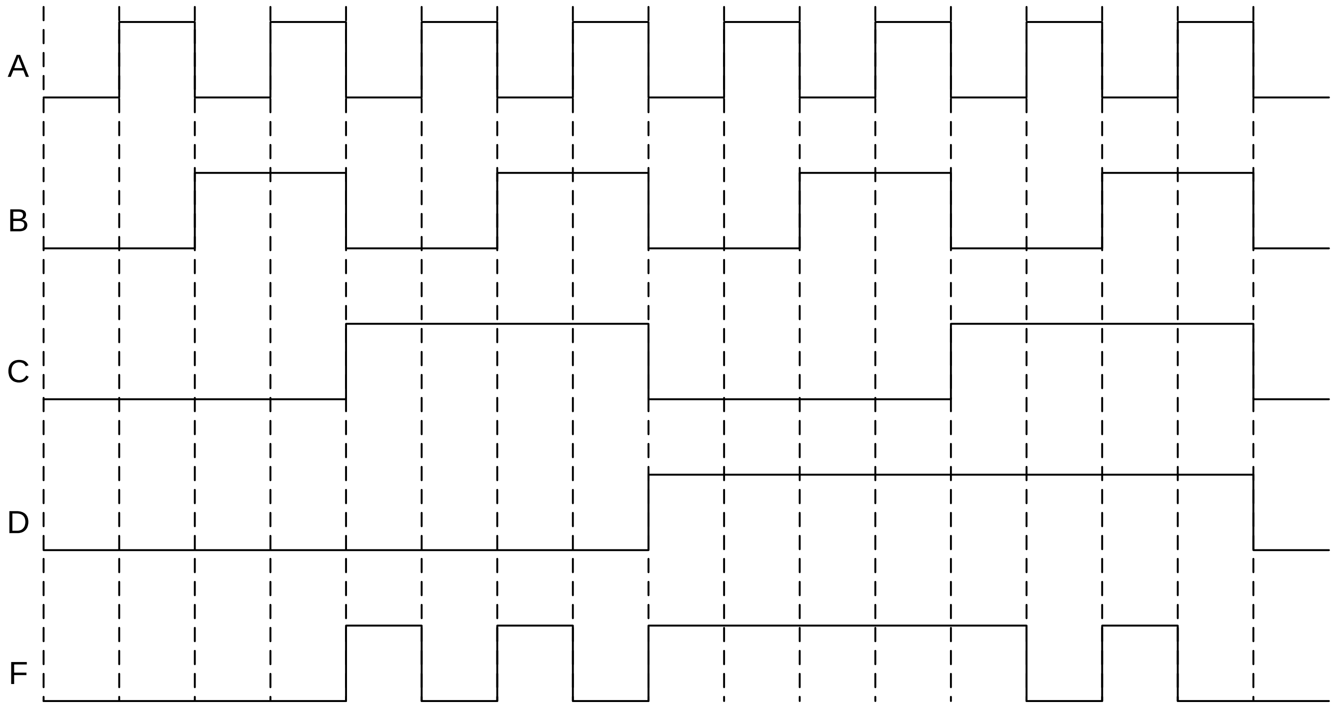
Exercise

Given the truth table, find the simplified SOP and POS form.

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Exercise

- Design two-level NAND-gate logic circuit from the follow timing diagram.



Don't care term

A	B	C	D	X
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	x
1	0	1	1	x
1	1	0	0	x
1	1	0	1	x
1	1	1	0	x
1	1	1	1	x

CD \ AB	00	01	11	10
00			X	
01			X	1
11			X	X
10			X	X

AD

Exercise

- **Design logic circuit that convert a 4-bits binary code to Excess-3 code**

A	B	C	D	W	X	Y	Z
0	0	0	0	0	0	1	1
0	0	0	1	0	1	0	0
0	0	1	0	0	1	0	1
0	0	1	1	0	1	1	0
0	1	0	0	0	1	1	1
0	1	0	1	1	0	0	0
0	1	1	0	1	0	0	1
0	1	1	1	1	0	1	0
1	0	0	0	1	0	1	1
1	0	0	1	1	1	0	0
1	0	1	0	x	x	x	x
1	0	1	1	x	x	x	x
1	1	0	0	x	x	x	x
1	1	0	1	x	x	x	x
1 1	1 1	1 1	0 1	x X	X X	X x	X x

Drawbacks of K-Map Technique

- Minimization is extremely complicated as the number of variables exceed 6
- It is a manual process and depends on the ability of the user

Characteristics of Quine-McCluskey

- It is able to handle large number of variables
- It does not depend on the ability of the user
- It gives us the minimized expression

Important Aspects

- Prime Implicant (PI)
  - *It is a group of minterms which cannot be combined with any other minterm groups*
- Essential Prime Implicant (EPI)
  - *It's a prime implicant in which one or more minterms are unique i.e. it contains at least one minterm which is not contained in any other prime implicant*

- The Quine-McCluskey technique consists of two parts:
  - First identify all the prime implicants by an exhaustive search
  - To identify the essential prime implicants and select from them the remaining prime implicants which can give the perfectly minimized expression

Example

- Simplify the following boolean expression using k-map and verify it using Quine-McCluskey method
  - Y(A, B, C, D) = $\sum m(0, 1, 3, 7, 8, 9, 11, 15)$

Solution using K-Map

CD \ AB	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	1	1	1
10	0	0	0	0

$\overline{B}\overline{C}$

CD

Minimized Expression: Y = $\overline{B}\overline{C}$ + CD

Solution using Quine-McCluskey

- First arrange all the minterms according to the number of 1's contained and form the group having no ones, one 1, two 1s, three 1s etc.

Group	minterm	A	B	C	D		
1	m0	0	0	0	0	√	Zero 1
2	m1	0	0	0	1	√	One 1
	m8	1	0	0	0	√	
3	m3	0	0	1	1	√	Two 1
	m9	1	0	0	1	√	
4	m7	0	1	1	1	√	Three 1s
	m11	1	0	1	1	√	
5	m15	1	1	1	1	√	fourth

- We compare each minterm in group n in which each minterm in group (n + 1)

- We should put a √ on each matched pair. A matched pair is a pair of minterms which differ only in one variable
  - E.g. m0 and m1, m0 and m8

Combination of minterms into groups of two

Group	Minterm (Matched pairs)	Binary Representation A	B	C	D	
0	m0 – m1	0	0	0	-	√
	m0 – m8	-	0	0	0	√
1	m1 – m3	0	0	-	1	√
	m1 – m9	-	0	0	1	√
	m8 – m7	1	0	0	-	√
2	m3 – m7	0	-	1	1	√
	m3 – m11	-	0	1	1	√
	m9 – m11	1	0	-	1	√
3	m7 – m15	-	1	1	1	√
	m11 – m15	1	-	1	1	√

Matched pairs of minterms

New terms generated from the matched pairs of minterms

- Now, we compare all the pairs of minterms in previous table with those in the adjacent groups, to find if we can form groups of four minterms.
  - E.g. the pair (m0 – m1) of group 0 is compared with each pair in group.
- Such matched pairs are ticked √ in the previous table

Group	Minterm (Matched pairs)	Binary Representation A	B	C	D	
0	m0 – m1 – m8 – m7	-	0	0	-	$\bar{B}\bar{C}$
	m0 – m8 – m1 – m9	-	0	0	-	
1	m1 – m3 – m9 – m11	-	0	-	1	$\bar{B}$D
	m1 – m9 – m3 – m11	-	0	-	1	
2	m3 – m7 – m11 – m15	-	-	1	1	CD
	m3 – m7 – m7 – m15	-	-	1	1	

Combination of minterms into groups of four

- Repeat the procedure for grouping. Hence it is seen that if the Quads of minterms in the adjacent groups of the previous table is grouped to obtain eight minterms.

- There are no such matchings in the previous table, hence the process of grouping will end here

- Collect all the nonchecked terms from all the tables derived. These are the prime implicants (PI) and they will be present in the simplified expression of Y.

- Therefore $Y = \overline{B}\overline{C} + \overline{B}D + CD$

PI	Decimal number corresponding to PI	Binary Representation							
		0	1	3	7	8	9	11	15
$\overline{B}\overline{C}$	0, 1, 8, 9	(X)	X			(X)	X		
$\overline{B}D$	1, 3, 9, 11		X	X			X	X	
CD	3, 7, 11, 15			X	(X)			X	(X)

Coverage Table

- The Essential Prime Implicant (EPI) are $\overline{B}\overline{C}$ and CD
- The final minimized expression

$$Y = \overline{B}\overline{C} + CD$$

Practice Question

- Minimize the following logic function using K-map and verify the answer using Quine-McCluskey method
  - $Y(A, B, C, D) = \sum m(0, 1, 2, 3, 5, 7, 8, 9, 11, 14)$